ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/2

PAPER 2

SPECIMEN PAPER 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae
- Electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer all questions in Section A and any five questions from Section B

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81\ \text{ms}^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (40).

Answer all questions in this section.

1 **(i)** If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx} - y\right)$ [3]

(ii) Find the Maclaurin expansion of the function $e^x \sin x$ as far as the term in x^3 by further differentiation of the result in **part (i),** [4]

2 **(i)** Find the equation of a circle which has the points $(-7;\ 3)$ and $(1;\ 9)$ as end points of its diameter. [3]

(ii) Hence or otherwise, find the equation of the tangent to the circle which passes through the point $(-7;\ 3)$. [4]

3 Prove by induction that

$$\sum_{r-1}^{n} ap^r = ap\left[\frac{1-pn}{1-p}\right]$$

for all positive integral values of n, where a and p are constants. [8]

4 By using the substitution $u = sinx$, show that

$$\int_0^{\frac{3n}{2}} \frac{cosx}{3+cos^2x}\,dx = \frac{1}{4}ln\frac{1}{3}$$ [8]

5 Let **H** be the set of all matrices of the form $\begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}$, where $y \in IR$.

Show that

(i) **H** does not form a group under matrix addition. [4]

(ii) **H** forms an abelian group under matrix multiplication.
[Assume associativity] [6]

Section B (80)

Answer any <u>five</u> questions from this section. Each question carries 16 marks.

6 **(a)** A plane, p, contains the point X with position vector $3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and the line, l, with vector equation

$\mathbf{r} = 2\boldsymbol{i} + \lambda(\boldsymbol{j} + \boldsymbol{k})$, where / is a parameter.

Find the **(i)** vector equation of p, [4]

(ii) shortest distance from the origin to p. [5]

(b) Two planes have equations

$$\boldsymbol{r} \bullet \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 3 \text{ and } \boldsymbol{r} \bullet \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} = 4.$$

find the **(i)** equation of the line of intersection of the two planes, [4]

(ii) angle between the two planes. [3]

7 **(a)** Given that matrix $\mathbf{T} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$,

(i) interpret geometrically the transformation represented by T, [2]

(ii) find the image of a line $y = 2x + 3$ under the transformation T, giving your answer in the form $ax + by + c = 0$.

[3]

(b) **(i)** Given that matrix $\mathbf{N} = \begin{pmatrix} 1 & -1 & 0 \\ 5 & -4 & 1 \\ 2 & 2 & 1 \end{pmatrix}$,

find N^{-1}, the universe of matrix N. [6]

(ii) Hence, or otherwise solve the simultaneous equations:

$$\begin{aligned} x - y &= -4 \\ 5x - 4y + z &= -12 \\ 2x + 2y + z &= 11 \end{aligned}$$

[5]

[Turn over

8 Given the points A(0; 2; 2), B(4; 1; 0) and C(–2 ; 0, 3).

Find the

(i) value of q, given that the Cartesian equation of a perpendicular bisector of a line joining the points A and B, passing through point D (1; q; 0) is $x - 2 = \frac{2y-3}{4} = z - 1$. [5]

(ii) exact shortest distance of the line in part **(i)** to the origin, [5]

(iii) Cartesian equation of the plane containing points A, B and C. [6]

9 The temperature of meat in an oven is q. The rate of increase of the temperature of the meat in the oven is proportional to the difference in the temperature that exists between the meat and the oven. The oven is kept at a constant temperature of 100 °C at any instant.

(i) Show that $\frac{dq}{dt} = k(100 - q)$, where k is a constant. [3]

(ii) A piece of meat which was initially at 4°C is placed into the oven at time $t = 0$ seconds. After one minute, the temperature rose to 16°C.

Express q in terms of t seconds in its simplified form. [8]

(iii) Find q when t = 2.5 minutes. [2]

(iv) Find the time it takes for temperature of meat to rise to 80ºC. [3]

10 **(a)** Given that **z** $= \frac{5+i}{2+3i}$, find the fifth roots of **z** in the form re^{iq}. [8]

(b) Given that $1+i$ is a root of the equation $z^3 + pz^2 + qz + 6 = 0$ where p and q are constants,

find the

(i) other **two** roots, [4]

(ii) values of p and q. [4]

11 It is given that $y = x^2e^x$.

(i) Find $\dfrac{d^2y}{dx^2}$. [5]

(ii) Show that $\frac{d^3y}{dx^3} = 4e^x + 2xe^x + \frac{d^2y}{dx^2}$. [3]

(iii) Hence, prove by induction that the statement

$\frac{d^xy}{dx^n} = (n-1)(2e^x) + 2xe^x + \frac{d^{n-1}y}{dx^{n-1}}$ is true for all positive integers, n,

such that $n \geq 2$. [8]

12 **(a)** The 4th term of an arithmetic progression is 42 and the sum of the first three terms of the series is 12.

Find the

(i) first term and the common difference, [4]

(ii) sum of the first twenty terms. [4]

(b) The 3rd term of a geometric progression is 36 and the 5th term is 16.

Find the

(i) first term and the common ratio, r, given that $r < 0$, [4]

(ii) sum to infinity of the series. [4]

BLANK PAGE

BLANK PAGE

BLANK PAGE